RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. FOURTH SEMESTER EXAMINATION, AUGUST 2021
SECOND YEAR [BATCH 2019-22]

Date :13/08/2021 — **MATHEMATICS(General)**
Time:11 am - 1 pm — **Paper** : IV — Full Marks : 50

Instructions to the students

- Write your **College Roll No, Year, Subject** & **Paper Number** on the top of the **Answer Script**.
- Write your **Name, College Roll No, Year, Subject** & **Paper Number** on the **text box of your e-mail**.
- Read the instructions given at the beginning of each paper/group/unit carefully.
- Only handwritten (by blue/black pen) answer-scripts will be permitted.
- Try to answer all the questions of a single group/unit at the same place.
- All the pages of your answer script must be numbered serially by hand.
- In the last page of your answer-script, please mention the total number of pages written so that we can verify it with that of the scanned copy of the script sent by you.
- For an easy scanning of the answer script and also for getting better image, students are advised to write the answers in single side and they must give a minimum 1 inch margin at the left side of each paper.
- After the completion of the exam, scan the entire answer script by using Clear Scan: Indy Mobile App OR any other Scanner device and make a **single PDF file (Named as your College Roll No)** and send it to [redacted]

Group - A

(Ordinary Differential Equation)
(All the symbols have their usual meaning.)

Answer any 3 out of 5 questions.

1. Solve: $(x+3)^2\frac{d^2y}{dx^2} - 4(x+3)\frac{dy}{dx} + 6y = x$. [5]
2. Solve: $(D^2 - 2D + 1)y = e^{3x}\cos x$. [5]
3. Solve: $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = e^x + x^2$. [5]
4. Find the equation of the orthogonal trajectory of the family of curves $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$, a being the parameter. [5]
5. Obtain the complete primitive and the singular solution of $py = p^2(x-b) + a$. [1+4]

Group - B

(Calculus)

(All the symbols have their usual meaning.)

Answer any 2 out of following 3 questions.

6. Show that $\{(\frac{10}{11})^n\}$ is convergent and converges to 0. [5]

7. Examine the convergence of $\sum\limits_{n=1}^{\infty} \frac{1}{(2n-1)(2n+1)}$. [5]

8. Examine the convergence of $\sum\limits_{n=2}^{\infty} (\frac{1}{\sqrt{n}} + \frac{1}{\sqrt{n-1}})$. [5]

Answer any 5 out of following 7 questions.

9. (a) Define continuity of a real-valued function at a point. [1]

   (b) Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = \begin{cases} 0 & \text{for} \quad x \in \mathbb{R} \setminus \mathbb{Q}. \\ 1 & \text{for} \quad x \in \mathbb{Q}. \end{cases}$. Show that f is not continuous at any $x \in \mathbb{R}$. [4]

10. Prove that the radius of curvature of the curve $x^3 + y^3 = 3axy$ at the point $(\frac{3a}{2}, \frac{3a}{2})$ is equal to $\frac{3a}{16}\sqrt{2}$. [5]

11. Find the asymptotes of the following curves:

    (a) $x^3 + y^3 = 6x^2$. [2.5]

    (b) $y = 2\sqrt{x^2 + 4}$. [2.5]

12. Using Beta and Gamma function, find the value of the following integral:

    (a) $\int_0^1 x^{-\frac{2}{3}}(1-x)^{-\frac{1}{3}}dx$. [2]

    (b) $\int_0^{\frac{\pi}{2}} \sin^5\theta \cos^5\theta d\theta$. [3]

13. (a) Find the area of the surface generated by revolving about the Y-axis that the part of astroid $x = a\cos^3\theta, y = a\sin^3\theta$ that lies in the first quadrant. [2]

    (b) Find the volume of the solid generated by revolving the cardioid $r = a(1-\cos\theta)$ about the initial line. [3]

14. If $y = (\sin^{-1} x)^2$, using Leibnitz's rule prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2y_n = 0$. [5]

15. Using Taylor's theorem, prove that $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots \infty$. [5]